Lecture : Prof. Patrick H. Diamond

Notes : Jiacong Li

Rotation Theory

- History approaches & Where we are.

→ Particles conserved.
Momentum can exchange between particles and waves.

→ Q.L.T. : By quasi-linear theory, these are conserved:

∂_t [Resonant particles' kinetic energy density + wave energy density] $=0$

∂_t [Particle (resonant + nonresonant) energy density + field energy density] $=0$

∂_t [Particle momentum density] $=0$

Non-resonant particles, field momentum density $=0$

→ Wave momentum flux

$$\underline{\underline{\Pi}} = \int d\underline{k}\, \underline{V}_{gr}\, \underline{k}\, N$$

N → Distribution density

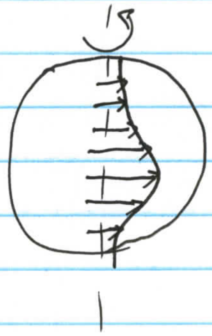
In general, mean field theory for $\underline{\underline{\Pi}}$

→ Comparison

	Solar Differential Rotation	Tokamak Intrinsic Rotation
Drive	Fusion → heat flux	heating → ∇P, ∇T
$\underline{\underline{\Pi}}$	Convection	Drift wave turbulence
Symmetry Breaking	Ω, etc...	$\langle \underline{B} \rangle$ structure, radial profile, ..
	solar wind	separatrix, SOL

→ Rotation profile.

(▷ Rotation profile for Sun :



▷ Heating → flux ~~drift~~ driven turbulence → $\langle \underline{\tilde{V}} \underline{\tilde{V}} \rangle$

→ Mean field theory for $\langle \underline{\tilde{V}} \underline{\tilde{V}} \rangle$
↓
transport coeffs + mean quantities.

→ Other sources of rotation

$$n \frac{\partial}{\partial t} \langle V_\phi \rangle = - n \underline{\nabla} \cdot \underline{\underline{\Pi}} + \underbrace{S_T}_{\text{parallel acceleration (not clear)}}$$

(L. Wang and P.H. Diamond, P.R.L. 2013)

- Reynolds stress

$$\underline{\underline{\Pi}}_{r,\phi} = -\chi_\phi \frac{\partial \langle v_\phi \rangle}{\partial r} + \overset{\nearrow \text{pinch}}{V} \langle v_\phi \rangle + \Pi_{resid}$$

① $\chi_\phi \sim \chi_i$, diffusion coeff.

② V : pinch , $V = V_{TEP} + V_{Thermo}$

Turbulent Equipartition Pinch

TEP of density : $\frac{d}{dt}\left(\frac{n}{B}\right) = 0$, $\Gamma = -D\left(\frac{\nabla n}{B} - \frac{n}{B}\underbrace{\frac{\nabla B}{B}}_{\text{pinch}}\right)$

TEP of angular momentum density : $\frac{d}{dt}\left(\frac{n v_\parallel R}{B^2}\right) = 0$,

$$V_{TEP} = n v_\parallel \nabla\left(\frac{R}{B^2}\right)$$

③ Π_{resid} : Driven by ∇T, ∇P, ∇n

$\nabla \cdot \Pi_{resid}$ ⇝ local intrinsic torque

independent of $\langle v_\phi \rangle$ ⇝ Spin up the plasma from rest ($\langle v_\phi \rangle = 0$)

Acting with boundary condition.

$$\frac{\partial}{\partial t}\langle v_\phi \rangle = -\frac{\partial}{\partial r}\left(-\chi_\phi \frac{\partial \langle v_\phi \rangle}{\partial r} + V\langle v_\phi \rangle + \Pi_{resid}\right)$$

$\langle v_\phi \rangle \equiv 0$ ↪ $\partial_t \int_0^a dr \langle v_\phi \rangle = -\Pi_{resid}\Big|_0^a$

- Symmetry breaking

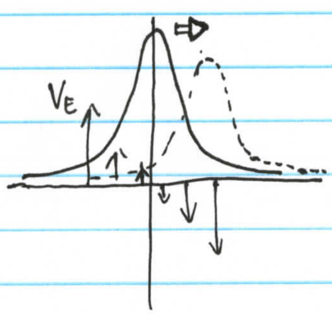
→ $\langle k_\theta k_\parallel \rangle \sim \sum_{\underline{k}} k_\theta k_\parallel |\phi_{\underline{k}}|^2 = 0$ if absent symmetry breaking

→ Requires symmetry breaking

- Convert radial inhomogeneity into paralle ~~spectral~~ spectral asymmetry

- Mechanisms : $\left\{ \begin{array}{l} \text{Electric field shear } \langle V_E \rangle' \\ \text{Intensity gradient } \partial_r I \end{array} \right.$

$\langle V_E \rangle'$: Shift modes off resonant surfaces



$\Longleftrightarrow$ spectral shift

Note : Problem ! $\langle V_E \rangle'$ ~~turn~~ may turn instability off.

$\partial_r I$: $|\phi_{\underline{k}}|^2 \cong |\phi_{\underline{k}}(r_0)|^2 + (r - r_0) \frac{\partial}{\partial r} |\phi_{\underline{k}}(r_0)|^2$

$\Pi_{r,resid} = \left\langle \frac{k_\theta^2}{L_s} \Delta^2 \right\rangle \frac{\partial}{\partial r} |\phi_{\underline{k}}|^2$

Profile curvature $\Longrightarrow$ $\partial_r I$ To show this, consider

constant total heat flux , $Q = -(\chi_T + \chi_{neo}) \partial_r \langle T \rangle$

$$Q' \cong 0 \Rightarrow \frac{1}{\chi_T}\partial_r \chi_T \cong -\frac{1}{\partial_r\langle T\rangle}\partial_r^2\langle T\rangle - \frac{1}{\chi_T}\frac{\partial \chi_{neo}}{\partial r}$$

Assume $\chi_T > \chi_{neo}$,

$$\frac{\partial_r I}{I} \sim \frac{\partial_r \chi_T}{\chi_T} \cong -\frac{\partial_r^2\langle T\rangle}{\partial_r\langle T\rangle} \rightarrow \text{Profile curvature.}$$

- Reversals

→ Wave momentum flux

$$\Pi_{r,\parallel}^{wave} = \int d\underline{k}\, k_\parallel \left\{ -\tau_{c,\underline{k}} V_{gr}^2 \frac{\partial\langle N\rangle}{\partial r} + \tau_{c,\underline{k}} \boxed{V_{gr}} k_\theta \langle V_E\rangle' \frac{\partial\langle N\rangle}{\partial k_r} \right\}$$

Diffusion of wave momentum by short mfp profile

Refraction

$$V_{gr} \sim \frac{2 k_\theta V_r V_*}{(1 + k_\perp^2 \rho_s^2)^2}$$

Changing sign in V_{gr} → Rotation reverses

→ TEP → ITG , mode flip

→ Mode propagation direction flip

→ Reversal.

References:

[1] P.H. Diamond, etc., Nucl. Fusion 53 (2013) 104019

[2] L. Wang, P.H. Diamond, PRL 110, 265006 (2013)

[3] P.H. Diamond, "Intrinsic Rotation and Toroidal Momentum Transport: Status and Prospects". presentation at USTC